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User's Manual for Soils and Aggregates Sample Testing (SASTP) Computer Program

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Abstract	The Soils and Aggregates Section of the Materials Engineering and Research Office, has been conducting the proficiency sample-testing program for aggregates and soil tests, since the late 1980's. A computer program that was developed to run on Disk Operating System (DOS) was used for analysing the data from this sample-testing program. The computer network system used by the ministry no longer supports the programs that were developed to run on DOS, and as a result, a new program that is compatible with WIN 2000 has been developed. The new program has several added features compared to the old program and is based on statistical criterion widely accepted and recommended by agencies such as American Society for Testing Materials. In addition, the program has three different statistical methods to identify and reject outlying observations. The intent of this manual is to provide details of the statistical techniques used in the program and to aid the users in operating the program. In addition, relevant statistical tables and numerical examples are also provided for anyone who is interested in checking the analytical results manually.
Key Words	Critical value, iterations, mean, outliers, standard deviation, standardised deviate, statistical techniques
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User's Manual for Soils and Aggregates Sample Testing (SASTP) Computer Program

December 2004

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Executive Summary

The Soils and Aggregates Section of the Ministry of Transportation, Ontario was using a computer program that was developed in early 1980's to run on Disk Operating System (DOS) for analysing the data from its annual Proficiency Sample Testing Program. The computer network system of the ministry no longer supports the programs that were developed to run on DOS, and the system has been upgraded to operate on WIN 2000. As a result, a new program, SASTP, has been developed. In addition, provision was made to accommodate more laboratories and to include additional tests in the program.

The new program is based on a statistical criterion that is recommended by agencies such as the American Society for Testing Materials (ASTM) for removing outlying observations, and has three different options to analyse the data. The program has several added features compared to the old program (LABCOS) and in particular, the program is capable of analysing up to 500 participating laboratories and 21 test methods with provisions to add more test methods to its database.

The purpose of this manual is to explain the statistical techniques used in developing the program and to provide an operational guide to use of the software. The statistical process involved in identifying and removal of outliers from the sample population are explained in detail in Chapter 2, with numerical examples. The operational guidance to the program is provided in Chapter 3. In addition, relevant statistical tables are provided to check the calculations manually.

1.0 Introduction

The Soils and Aggregates Section of the Materials Engineering and Research Office have been conducting a proficiency sample-testing program since the late 1970's. The annual sample-testing program is primarily intended to provide a means for the laboratories used by MTO to see if they are performing satisfactorily and to qualify these laboratories to perform quality control and quality assurance testing for MTO contracts. The participants in this annual proficiency program include the MTO laboratory, municipalities, and private sector laboratories owned by contractors, aggregate producers, and engineering consultants. The Canadian Council of Independent Laboratories (CCIL) also uses the data gathered by this sample-testing program to evaluate and certify aggregate testing laboratories.

The final report issued to the laboratories contains ratings for each test method. The ratings are based on the standardised deviate computed from the mean and standard deviation determined after removing the outliers. The ratings for each test method are also used to calculate an overall laboratory rating to measure the overall performance of the laboratories. This rating system has acted as an incentive for laboratories to improve their performance and is used as a guide to select highly proficient laboratories. The rating of individual tests is also used to assess the competence of laboratories applying for registration to do work for MTO.

A computer program (LABCOS) that was developed in the late 1970's was used for the statistical analysis of test results submitted by the participants. This computer program was written in FORTRAN and ran on DOS (Disk Operating System). The ministry has upgraded all of its computer systems to operate on WIN 2000, and the services are no longer available for operating the programs that were developed to run on DOS. As a result, MTO has developed a new program that is compatible with WIN 2000.

The new program is based on established statistical criteria for rejecting outlying observations. An outlier is an observation that appears to deviate markedly from the sample population or is a measurement that, for a specific degree of confidence, is not part of the population. Various statistical methods are available to test the hypothesis that the suspect observations are not outliers. The new program, SASTP, has two options to detect outlying observations or outliers in a set of data: namely, the Critical Value Method and the Iterative Technique (Jack Knife), which was also available in the old program (LABCOS). These techniques are based on two different statistical methods and yield confidence intervals depending on the number of laboratories participating in a particular test (number of observations) and the distribution of test data.

The Critical Value Method is one of the methods recommended in ASTM E 178 for single samples. The critical values published in ASTM E 178 are based on the recommendations of

Grubbs (1969 and 1972) for the selection of outliers, and are limited to 147 observations only. This will not be useful for our purposes, because over 200 laboratories participate in our annual Proficiency Sample Testing Program. The critical values for the development of the new program SASTP have been calculated at five percent significance level using the recommendations by Grubbs (1969 and 1972) for identifying outliers. For the Critical Value Method, the computer will reject results, which are significant at the 5% level.

In the case of Jack-Knife technique, the significance level will depend on the number of observations and the level of significance may vary considerably. For this reason, there are possibilities for the program to reject results at a confidence level lower than 95%. In view of this, the users of this program are advised to employ the Jack Knife method of analysis with caution.

This manual contains the instructions for the use of the Soils and Aggregate Sample Testing Program (SASTP) developed by the Engineering Standards Unit, Highways and Business Services Application Solutions Branch (HBSASB) of the ministry. The program, SASTP, has several features that include creation of excel spreadsheet for generating scatter diagram using the "Grapher" plot program, participants' list with identification of tests in which a particular lab has participated, and a scatter plot with statistical data and outlier labs for the individual test. The Microsoft Excel spreadsheet generated at the end of the analysis of a particular test includes participant's test results, standardised deviate, rating for each sample, and the identification of the outlier labs. The program also permits printing of individual lab results for mailing purposes.

The SASTP program was used for the analysis of 2003 Proficiency Sample Testing data and preparation of the final report. The information provided in the Aggregate and Soil Proficiency Sample Testing Report for 2003 (MERO-008, January 2004) is an example for the output options that are available in this program.

2.0 Statistical Techniques

2.1 LABCOS COMPUTER PROGRAM

The ministry has been using the computer program, LABCOS, which was developed in the late 1970's for analysing the data from its annual Proficiency Sample Testing Program. LABCOS program had the analytical options of using Three Standard Deviation Technique or the Iterative (Jack Knife) Technique to reject outlying observations or outliers in a set of data. These techniques are based on two different statistical methods and yield confidence intervals depending on the number of observations (number of laboratories participating in a particular test method) and the distribution of data.

The three Standard Deviation technique used in LABCOS is not based on a statistically acceptable criteria for identifying outliers. The second option, Jack Knife method, in LABCOS is based on statistical criteria. However, the confidence interval or the significance level in this method may vary depending on the number of observation and the nature of data distribution.

In the Three Standard Deviation Technique, the doubtful observation is included in the calculation of mean and standard deviation of the sample population, i.e., data set. The absolute values of standardised deviate for all the observations in the population are then calculated and the largest standardised deviate is compared with a numerical value of three. The observation with the largest standardised deviate is rejected or identified as an outlier if the standardised deviate is higher than three. One outlier at a time is detected and rejected. The outlier is removed from the data set and the iterations are continued until no outliers are detected. A revised mean and standard deviation are calculated after deleting each outlier. The ratings of the laboratories are determined based on the revised mean, standard deviation and standardised deviate calculated from the revised standard deviation.

The outlying observations must be identified based on an established statistical criterion, including a confidence interval for the probability of occurrence. The Three Standard Deviation technique does not use any statistical criteria for testing outliers or test the hypothesis that the rejected laboratories are not outliers. Further, the outliers are not based on a predetermined confidence interval. The confidence interval in this procedure will depend on the distribution of data and the number of observations, i.e. number of participating laboratories. For these reasons, this technique may identify outliers erroneously, i.e., the laboratories that should have been rejected based on a statistical criteria may be accepted when the number of participating laboratories are less than about 30 for a significance level of 5%, and vice versa when the number of observations are higher than 30.

2.2 OUTLYING OBSERVATIONS

In a set of data or test results, an outlier is one that appears to deviate markedly from the sample population. An outlier may be merely an extreme manifestation of the random variability inherent in the data or may be the result of gross deviation from the prescribed experimental procedure, calculation errors or error in reporting. In dealing with suspected outlying observations or outliers, our purpose is to remove those observations that do not belong to the sample population and to provide some statistical criteria for doing so.

Various statistical methods are used to test the hypothesis that the suspected observations are not outliers. In most of these methods, as ASTM E178 states, "the doubtful observation is included in the calculation of the numerical criterion (or statistic), which is then compared with a critical value based on the theory of random sampling to determine whether the doubtful observation is to be retained or rejected". The critical value is that value of the sample criterion that would be exceeded by chance with some specified (small) probability on the assumption that all observations did indeed constitute a random sample from a common system of causes, a single parent population, distribution or universe.

2.3 STANDARDISED DEVIATE

The standardised deviate is a measure of how many standard deviations (s) an individual test result is away from the mean ($\overline{X}$) or the average value of the population. It is obtained by: subtracting the mean from the actual test result reported by the laboratory, and dividing by the standard deviation. The mean and standard deviation used in computing standardised deviates are calculated after deleting the outliers. If an individual test result is less than the mean, the standardised deviate is negative, and if the test result is higher than the mean, the standardised deviate is positive. The standardised deviate is calculated as follows:

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} \qquad s = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{N-1}}$$

$$\text{Standardized Deviate} = \frac{(X_i - \overline{X})}{s}$$

Where:

$\overline{X}$	-	Arithmetic mean or average of the population
X_i	-	Individual lab test result in a data set
N	-	Number of observations or participants in a particular test
s	-	Standard deviation of the population

2.4 CRITICAL VALUE METHOD

The Three Standard Deviation Technique is replaced in the SASTP program with the Critical Value Method, which is recommended in Section 4 of ASTM E 178 for rejecting outliers. The critical value is that value of the sample criterion, which would be exceeded by chance with some specified probability on the assumption that all observations in the sample come from the same normally distributed population. The critical values provided in the Table 1 of ASTM E 178 are based on Grubbs (1972) and are limited to 147 observations only. As of 2003, over 200 laboratories participate in our annual Proficiency Sample Testing Program and the number of participants may increase in the future. In anticipation of this, provisions to handle 500 observations are made in the SASTP program. The critical values for the development of the SASTP program were calculated based on Grubb's test (1969 and 1972) to reject outliers at a significance level of 5%.

Grubbs' test (Grubbs 1969 and 1972) is based on the assumption of normality and uses Student's "t" distribution for computation of critical values. Similar to other methods, the outlier in this method is also removed from the data set and the test is iterated until no further outliers are detected. However, the multiple iterations may change the probabilities of detection, and the test should not be used for small numbers of observations (six or less), since it frequently tags most of the points as outliers.

The Grubb's test statistic is defined as

$$\mathbf{G} = \frac{\left|\left(X_i - \overline{X}\right)\right|}{s}$$

The hypothesis of no outliers is rejected if

$$\mathbf{G} > \frac{(N-1)}{\sqrt{N}} \sqrt{\frac{t^2_{(\alpha/2N),N-2)}}{N-2+t^2_{(\alpha/(2N),N-2)}}}$$

Where:

N	-	Number of observations in a dataset
α	-	Significance level
G	-	Largest absolute deviation from the sample mean in units of the sample standard deviation, i.e., the largest standardised deviate
X_i	-	Individual lab test result in the data set
$\overline{X}$	-	Arithmetic mean or average of the data set
s	-	Sample standard deviation

Example 1: Calculation of Critical Value "G_N"

The critical value is computed for thirteen (13) observations at a confidence level of 95%:

Number of Observations, N = 13
Significance level, α = 5%

$$G = \frac{(N-1)}{\sqrt{N}} \sqrt{\frac{t^2_{(\alpha/2N),N-2)}}{N-2+t^2_{(\alpha/(2N),N-2)}}}$$

$\alpha = 0.05$; $\Delta\alpha = \alpha / 2N = (0.05)/(2 \times 13) = 0.001923$

$(1 - \Delta\alpha) = 1 - 0.00192 = 0.9981$; $(N-2) = 11$

$t_{\{\alpha/2N, (N-2)\}} = t_{(0.9981, 11)}$

From probability distribution table in Appendix B

$t_{(0.9981, 11)} = 3.81795$

(N-1)/sqrt(N) = (12)/ sgrt(13) = 3.3282

G = (3.3282) (3.81795)/sqrt{11 + $(3.81795)^2$} = 2.512

$G_{13} = 2.512$

The critical value G_{13} calculated in the example is based on the 't' Distribution table provided in the Appendix B, and is slightly less than that of G_{13} in Appendix A, calculated using the Microsoft Excel. The 't' distribution values used in the SASTP program are linked to a commercially available statistical component Entisoft Tools V2.0.

In the Critical Value method, the doubtful observation is included in the calculation of mean and standard deviation of the population. The standardised deviate for all the observations are then computed and the largest value (T_N) from the population is compared with the critical value (G_N) at 95% confidence level corresponding to the number of observations. One outlier at a time is identified and rejected similar to that of the Three Standard Deviation technique. The outlier is removed from the data set and the iterations are continued until no outliers are detected, and a revised mean and standard deviation are calculated after deleting each outlier. The ratings of the laboratories are determined based on the revised mean, standard deviation and standardised deviate.

The test data from our proficiency sample program for Los Angeles Abrasion Loss (Test No. 8) is provided below to explain the procedures involved in the Critical Value Method.

Example 2: Identifying Outliers by Critical Value Method

Test Method: MTO LS-603 – Los Angeles Abrasion Loss

Test No.: 8

Number of Participants, N: 13

Lab. No.	X_i	$\overline{X}$	s	$X_i - \overline{X}$	$(X_i - \overline{X})/s$	$\lvert (X_i - \overline{X})/s \rvert$
19	49.800	49.894	7.199	-0.094	-0.013	0.013
31	58.500	49.894	7.199	8.606	1.195	1.195
35	53.600	49.894	7.199	3.706	0.515	0.515
38	48.500	49.894	7.199	-1.394	-0.194	0.194
47	53.120	49.894	7.199	3.226	0.448	0.448
56	49.500	49.894	7.199	-0.394	-0.055	0.055
80	54.200	49.894	7.199	4.306	0.598	0.598
98	49.100	49.894	7.199	-0.794	-0.110	0.110
101	48.300	49.894	7.199	-1.594	-0.221	0.221
105	50.200	49.894	7.199	0.306	0.043	0.043
122	51.800	49.894	7.199	1.906	0.265	0.265
188	54.000	49.894	7.199	4.106	0.570	0.570
292	28.000	49.894	7.199	-21.894	-3.041	3.041

The largest standardised deviate $T_{13} = 3.041$, which is higher than $G_{13} = 2.574$ in Appendix, and belongs to Lab No. 292. Therefore Lab No. 292 is identified as an outlier and rejected. The iteration is continued after removing Lab No. 292. The mean, standard deviation and standardised deviate (T_{12}) are calculated for twelve observations (N = 12) and compared with the critical value for G_{12}.

Lab. No.	X_i	$\overline{X}$	s	$X_i - \overline{X}$	$(X_i - \overline{X})/s$	$\lvert (X_i - \overline{X})/s \rvert$
19	49.800	51.718	3.055	-1.918	-0.628	0.628
31	58.500	51.718	3.055	6.782	2.220	2.220
35	53.600	51.718	3.055	1.882	0.616	0.616
38	48.500	51.718	3.055	-3.218	-1.053	1.053
47	53.120	51.718	3.055	1.402	0.459	0.459
56	49.500	51.718	3.055	-2.218	-0.726	0.726
80	54.200	51.718	3.055	2.482	0.812	0.812
98	49.100	51.718	3.055	-2.618	-0.857	0.857
101	48.300	51.718	3.055	-3.418	-1.119	1.119
105	50.200	51.718	3.055	-1.518	-0.497	0.497
122	51.800	51.718	3.055	0.082	0.027	0.027
188	54.000	51.718	3.055	2.282	0.747	0.747

The largest standardised deviate $T_{12} = 2.220$, which is less than $G_{12} = 2.519$ from Appendix A. Therefore, no lab is eliminated and the iterations are terminated. The ratings for the individual labs are assigned based on the standardised deviate computed for 12 observations.

2.5 ITERATIVE (JACK KNIFE) TECHNIQUE

The iterative technique was one of the options that were available in LABCOS to identify outlying observations. In some cases, the strict application of the critical value method tends to include laboratories in the population who are not doing a good job. In those cases, the application of the iterative (jack-knife) technique recommended by Manchester (1979) is used. In the iterative technique, the Constant C is computed using Fisher's F distribution. The Constant C depends on the number of participating laboratories, and is always higher than 2. In this technique, an outlying observation is rejected based on a statistical criterion but the confidence interval may vary depending on the number of participants and the distribution of the sample population.

In this method, after screening the data for any experimental or calculation errors, the doubtful observation is included in the calculation of mean and standard deviation of the sample population. The absolute residual values (actual test result minus the mean) are then calculated and the observation farthest from the mean by a unit of Cs (standard deviation multiplied by a Constant C) is identified as an outlier. One outlier at a time is identified and rejected similar to that of Critical Value Method. The outlier is removed from the data set and the iterations are continued until no outliers are detected. A revised mean and standard deviation are calculated after deleting each outlier. The rating of the laboratories are determined based on the revised mean, standard deviation and standardised deviate. The methods of computations and the procedure for identifying outliers are illustrated below.

a. Calculate the standard deviation "s" of the population using all the values in the sample.

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} \qquad s = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{N-1}}$$

b. Calculate the absolute residual value, $| X_i - \overline{X} |$ for each lab test data

c. Sort the residual values $|r_i| = | X_i - \overline{X} |$ in to an ascending order, $r_1, r_2, \ldots, r_N$

d. If $r_N > Cs$, reject the lab that correspond to r_N from the population and identify it as an outlier

e. Re-calculate the mean and standard deviation for the data set after removing the outlier and continue with the iteration until no observation is rejected

Where $C = \sqrt{\left\{\frac{3F}{1+\left(\frac{3F-1}{N-1}\right)}\right\}\left(\frac{N-1}{N}\right)}$

F – Value for 5% significant point in the Fisher's F Distribution Table (Appendix C) with a value of 3 and degrees of freedom N-1

The computations and procedures involved in identifying outliers by Jack-Knife method are explained using the test data provided under the critical value method in Example 2.

Example 3: Identifying Outliers by Jack-Knife Method

Calculate the mean and standard deviation using all the values in the population as in Step (a).

$\overline{X} = 49.894$; s = 7.199

Lab. No.	X_i	$X_i - \overline{X}$	$\lvert X_i - \overline{X} \rvert$
19	49.800	-0.094	0.094
31	58.500	8.606	8.606
35	53.600	3.706	3.706
38	48.500	-1.394	1.394
47	53.120	3.226	3.226
56	49.500	-0.394	0.394
80	54.200	4.306	4.306
98	49.100	-0.794	0.794
101	48.300	-1.594	1.594
105	50.200	0.306	0.306
122	51.800	1.906	1.906
188	54.000	4.106	4.106
292	28.000	-21.894	21.894

Lab No.	Sort $\lvert r_i \rvert$
19	0.094
105	0.306
56	0.394
98	0.794
38	1.394
101	1.594
122	1.906
47	3.226
35	3.706
188	4.106
80	4.306
31	8.606
292	21.894

Calculate C using Fisher's F Distribution Table in Appendix C for 95% confidence level:

Degrees of Freedom, N – 1 = 13 –1 =12; for value of 3, F = 3.49

$$C = \sqrt{\left\{\frac{3F}{1+\left(\frac{3F-1}{N-1}\right)}\right\}\left(\frac{N-1}{N}\right)} = \sqrt{\left\{\frac{3x3.49}{1+\left(\frac{3x3.49-1}{13-1}\right)}\right\}\left(\frac{13-1}{13}\right)} = 2.324$$

$Cs = (2.324)(7.199) = 16.730 < r_{13} = 21.894$

The largest residual value belongs to Lab No. 292, therefore eliminate this lab from the data set and re-calculate the mean and standard deviation.

Calculate the mean and standard deviation using the values in the population after removing Lab No. 292 as in Step (a).

$\overline{X} = 51.718;\ \ s = 3.055$

Lab. No.	X_i	$X_i - \overline{X}$	$\lvert X_i - \overline{X} \rvert$
19	49.800	-1.918	1.918
31	58.500	6.782	6.782
35	53.600	1.882	1.882
38	48.500	-3.218	3.218
47	53.120	1.402	1.402
56	49.500	-2.218	2.218
80	54.200	2.482	2.482
98	49.100	-2.618	2.618
101	48.300	-3.418	3.418
105	50.200	-1.518	1.518
122	51.800	0.082	0.082
188	54.000	2.282	2.282

Lab No.	Sort $\lvert r_i \rvert$
122	0.082
47	1.402
105	1.518
35	1.882
19	1.918
56	2.218
188	2.282
80	2.482
98	2.618
38	3.218
101	3.418
31	6.782

Same as before, calculate C for degree of freedom, N –1 = 11; F = 3.60

$$C = \sqrt{\left\{\frac{3x3.60}{1+\left(\frac{3x3.60-1}{12-1}\right)}\right\}\left(\frac{12-1}{12}\right)} = 2.288$$

$Cs = (2.288)(3.055) = 6.990 > r_{12} = 6.782$

The largest residual value $r_{12} = 6.782$ is less than Cs. Therefore no lab is eliminated and the iterations are terminated. The standardised deviate are computed from the mean and standard deviation obtained from the last round of iterations, and the ratings for the individual labs are assigned.

2.6 NO OUTLIER OPTION

The "No Outlier Option" was one of the options that were available in LABCOS is also included in SASTP. This option is required for preparation of preliminary report, which is usually issued in mid September to give feedback to the participants. At the stage of

preliminary report, the results are not screened for calculation or reporting errors, no outliers are identified and laboratory ratings are not assigned. In this option, the mean and standard deviation of the population are calculated and the standardised deviate for each test is provided for laboratories to examine their performance. The program will suppress the ratings of the labs, when this option is activated.

3.0 Operational Guide

3.1 PROGRAM FEATURES

3.1.1 Program Access

The SASTP program is available in the ministry computer network system, but the access to the program is restricted. Currently, the access is given only to two employees in the Soils and Aggregates Section, which is responsible for conducting the annual Soils and Aggregates Proficiency Sample Testing Program. Application to have access to the program should be approved by the Manager of Soils and Aggregates Section and submitted to Highways and Business Services Application Solutions Branch (HBSASB).

The program is accessible through a desktop of the authorised staff, and is identified by an icon "**sastp.mdb**". The main screen with the name of the program, Soils and Aggregates Sample Testing Program, as shown on Figure 1 will appear when double clicking on the icon. The control buttons and the operations are described below.

3.1.2 Sets in SASTP

Set: All the information related to the year of the Proficiency Sample Program is stored in a set. A set represents the data processing and analytical functions associated with the program. These functions include, data processing, performing analyses and generating individual lab test results to report to the participants. The sequence of operations involved in a set is as follows:

- Creating a set with a unique name;
- Selection of laboratories participating in the program;
- Entering data when the test results are submitted by participants;
- Selection of a particular test for the statistical analysis;
- Statistical analysis of data for rejecting outlier and to assign ratings;
- Generating individual lab results upon completion of statistical analyses; and
- Printing of individual lab test reports.

New Set: To begin a new set, click the button **"New"**, shown on the main screen (refer to Figures 1 and 2). The screen will prompt and ask to enter the name of the new set. If there is no new set to create, simply exit by clicking the "**cancel**" button. The set will consists of the following:

- Set name usually identified by the year of the program
- Set dates can be entered in any form and it will be reformatted automatically
- All the data related to the year of the program is stored in a set

Delete a Set: Click the "**Delete**" button to delete a set. When this operation is executed, all the information in a set will be deleted. For this reason, the computer will prompt with a warning of possible deletion of information before actual deletion.

Selecting a Set: To select a set from the previous correlation programs or access to the data from the past programs, use the combo box with the set name and click on the arrow. It will display a list of all the sets. Select the set required by clicking on it.

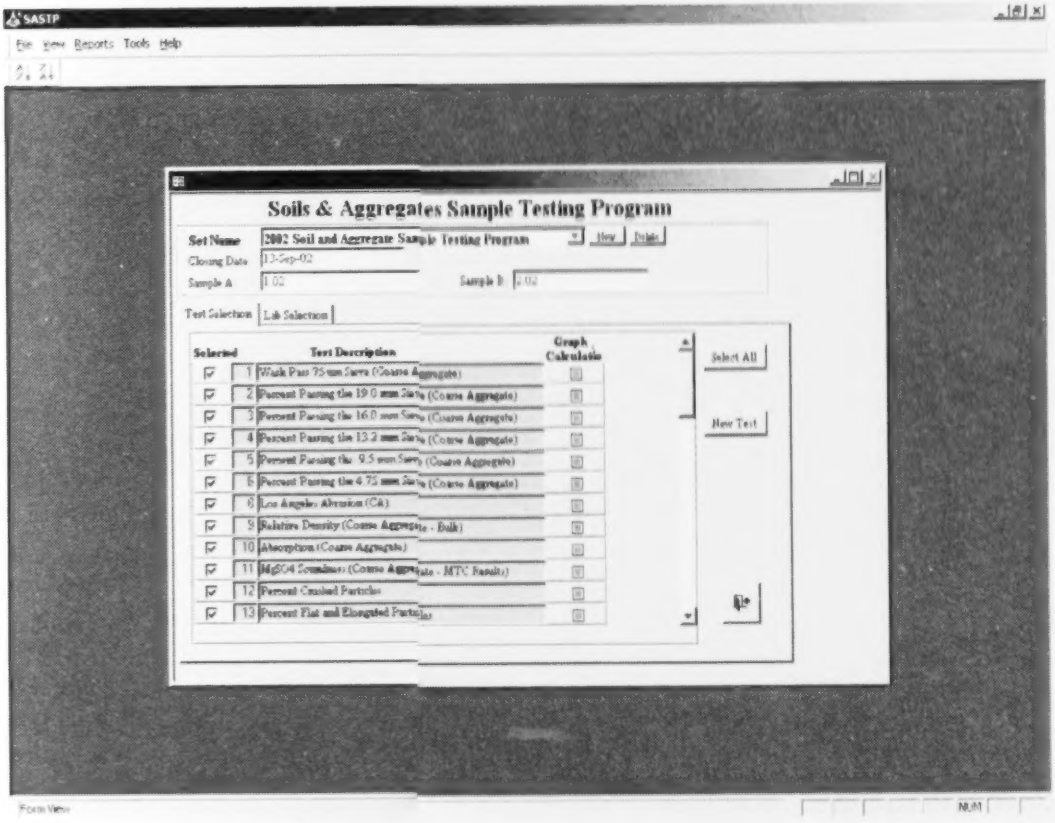


Figure 1 Display of Main Screen

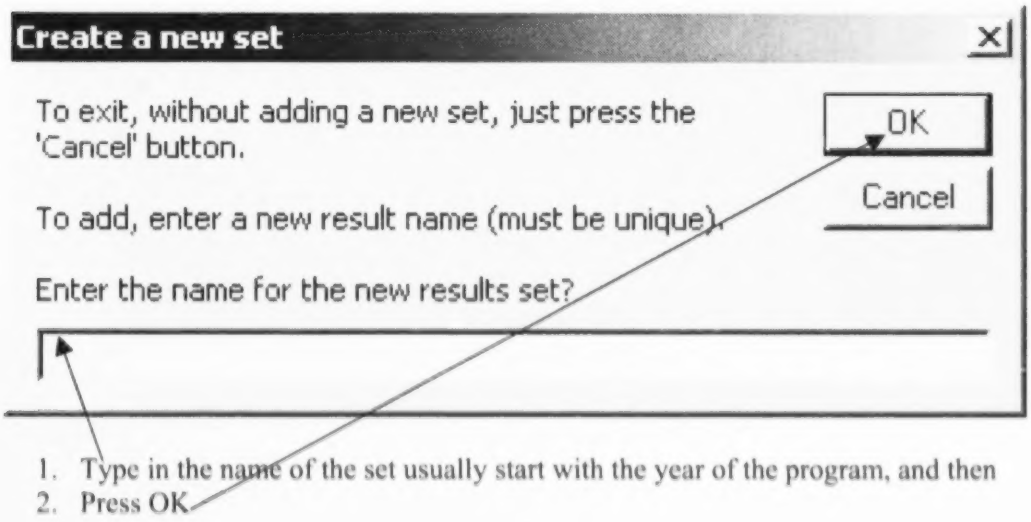

Create a new set
To exit, without adding a new set, just press the 'Cancel' button.
To add, enter a new result name (must be unique).
Enter the name for the new results set?
OK
Cancel
1. Type in the name of the set usually start with the year of the program, and then
2. Press OK

Figure 2 Computer Prompt for Creating a New Set

3.1.3 Laboratory Information

Editing Existing Labs: To edit the contact, company name, phone number, fax number or e-mail address of a lab in the database, double click on the laboratory in question from the main screen. The program will display the laboratory details. Make changes as required, the program will save the changes automatically when the window is closed.

New Laboratory: To add a new lab to the database, click the "**Lab Selection**" button on the main screen. This will display two additional buttons "**Select All**" and "**New Lab**" on the main screen. Click the "**New Lab**" button and enter the information related to the new lab in the area displayed. The lab number is unique and confidential, and is automatically assigned in the order in which the entries are made. The program will never re-use a lab number. If a lab is deleted, a new number will have to be assigned to the deleted lab.

Selection of Labs: "**Select All**" button indicated in Figure 1 is provided for a quick selection of all the labs in the database. If this is opted, the non-participant labs also will be included in the set and this procedure will take a few minutes to execute. Then the labs that are not participating will have to be disabled or removed from the set one by one by clicking on the box beside the name of the lab.

There are 323 laboratories in the database, but all of the laboratories may not participate in a particular year and also some of them are inactive for a long time. Altogether about 215 laboratories normally participate on an annual basis. It may be advisable to select the lab at the time of entering the data by clicking in the box besides the name of the lab.

Deletion of a Lab: When a lab is deleted by using "**Delete**" button, all the information associated with that lab including the information from the previous sets will be lost. To delete a lab, select the lab to be deleted from the main screen and double click on it. This will display the "**Laboratory Screen**" with a "**Delete**" button. A data loss warning will appear indicating possible loss of data. If the "**Yes**" button is chosen, the lab and the data associated with that lab will be deleted.

3.1.4 Editing Existing Tests and Addition of New Tests

Editing a Test in the Data Base: To edit a test in the database, click on the "**Test Selection**" button on the main screen. This will display all the tests in the database. Double click on the name of the test to be edited. This operation will retrieve all the information related to the test. When the test is retrieved for editing, it also retrieves all other tests in the same category (a category is group of similar tests – i.e.: Coarse Aggregates test using various sieves: 19 mm, 16mm, 13.2 mm, etc.) Enter the new or changed information about the test, a related test or its category. When finished, click the close window button. The information will automatically be saved. Make sure that the category number or coding numbers for the tests in the database are not changed. These numbers are linked to other operations of the program, and HBSASB of the ministry must be contacted if any changes are required.

New Test: Provision is allowed in the program to add new tests to the database. New tests can be added by accessing the File/New/Test and clicking the "**Test**" button. However, this operation requires a category number and coding number from the HBSASB. SASTP program is not ready for this operation and requires upgrading. Contact Mr. Bob Rutter of the Engineering Standards Unit, HBSASB at (905) 704 – 2153 for addition of new tests to the database.

3.2 TEST RESULTS INPUT, DATA ANALYSIS AND RATINGS

3.2.1 Test Results Input for Each Laboratory

The laboratories in the database can be sorted in the ascending or descending lab number order or in an alphabetical order. If the labs are to be sorted by lab number, place the cursor in the lab number box after selecting the "**Lab Selection**" and click the up or down arrow on the top left hand corner of the main screen. The test results submitted by the laboratories are entered in the following sequence:

- Click the "**Lab Selection**" button on the main screen;
- Scroll the cursor to locate laboratory by its number or name;
- Click the check box on the side of the laboratory tab under "**Data Enter**";
- The "**Laboratory Results**" screen as shown in Figure 3 will appear;
- Enter the results of the laboratory in question. If more than one set of results are

entered, the results will be averaged to get a single set of two samples figures to correlate; and

- Upon completion of entry, click the "**close button**", the results will be saved automatically.

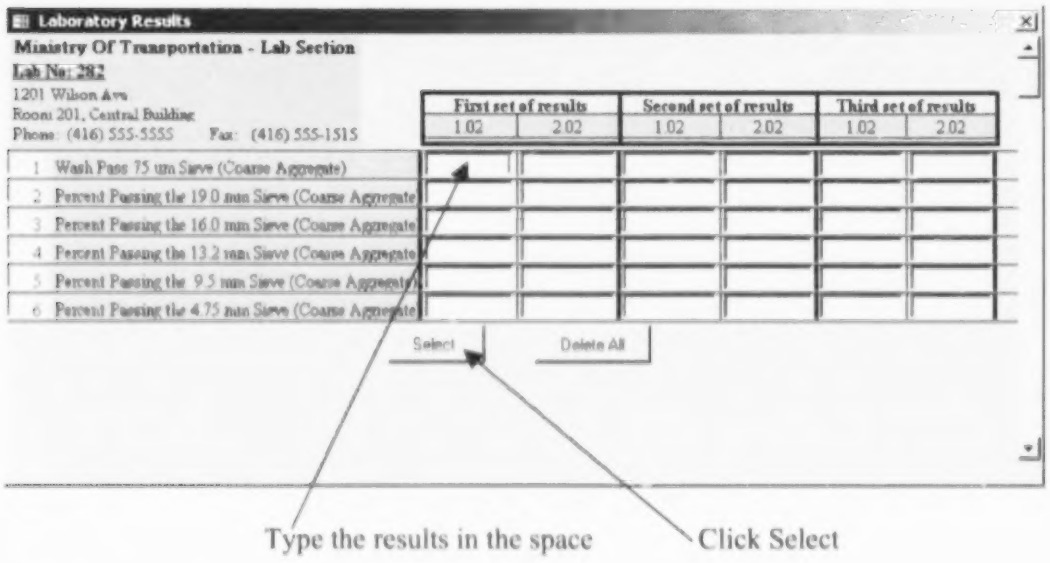

Laboratory Results
Ministry Of Transportation - Lab Section
Lab No: 282
1201 Wilson Ave
Room 201, Central Building
Phone: (416) 555-5555 Fax: (416) 555-1515
First set of results
Second set of results
Third set of results
1 02
2 02
1 Wash Pass 75 um Sieve (Coarse Aggregate)
2 Percent Passing the 19.0 mm Sieve (Coarse Aggregate
3 Percent Passing the 16.0 mm Sieve (Coarse Aggregate
4 Percent Passing the 13.2 mm Sieve (Coarse Aggregate
5 Percent Passing the 9.5 mm Sieve (Coarse Aggregate
6 Percent Passing the 4.75 mm Sieve (Coarse Aggregate
Select
Delete All
Type the results in the space
Click Select

Figure 3 Illustration of Test Results Entry

3.2.2 Data Analysis

As indicated in Section 2.0, the program has three options for the statistical analyses of data. To access these options, click the "**Test Selection**" button on the main screen. This will display the test numbers and the name of the tests in the database as shown in Figure 1. Scroll the cursor down to locate the test method for which the analysis is to be done. Click the check box on the side of the laboratory tab under "**Graph-Calculation**". This will display the screen with the title "**Calculate and Mark Outliers**" and other sub-titles such as "**Outlier Controls**", "**Test No**", and the analytical option selected or performed last. In addition, the screen will display the icons under the sub-title "**Outlier Control**" for the statistical analyses, type of output (graph or Excel spreadsheet), and option for manual removal of labs from the analyses. This screen displays confidential information of the participants and for this reason, the print of this screen is not included in this manual. Upon completion of the analyses, the screen will display the laboratory data, standardised deviate, ratings (Ranking), mean, standard deviation, and the outlier labs manually removed as well as the labs rejected through the analysis. This program should not be used if the number of observations is less than six. The statistical analyses are performed as follows:

- Click the "**Test Selection**" button on the main screen;
- Scroll the cursor to locate test number or name of the test method;

- Click the check box on the side of the laboratory tab under "**Graph-Calculation**";
- The "**Calculate and Mark Outliers"** screen will appear;
- Select the labs that are to be removed manually from the analysis by checking the box beside the lab name. No labs are removed from the analysis when "No Outlier" option is used. This option is used only for the preliminary report.
- Select the method of analysis is to be performed by clicking the appropriate icon, "**NO OUT**", "**CRIT VAL**", or "**JAC KNIF**";
- After the click on the analytical option, the computer will complete the computation for Sample A and prompt with "**OK**" sign and display the number of iterations. Click on the "**OK**" sign, the computer will repeat the procedure for Sample B.
- Upon completion of the analysis, the results can be exported in the form of Microsoft Excel Spreadsheet or viewed on the screen in the form of a scatter plot;
- Once the analysis is completed, click the "**close button",** the results will be saved automatically.

3.2.3 Test Method Ratings

The test method ratings, which are similar to the standardised deviate, but are in simple numeric form. The ratings for both samples along with other statistical data will be displayed for viewing. Ratings are determined as follows:

Rating 5 – data within 1.0 standard deviation of the mean;

Rating 4 – data within 1.01 to 1.5 standard deviation of the mean;

Rating 3 – data within 1.51 to 2.0 standard deviation of the mean;

Rating 2 – data within 2.01 to 2.5 standard deviation of the mean;

Rating 1 – data within 2.51 to 3.0 standard deviation of the mean; and

Rating 0 – data over 3.0 standard deviation of the mean.

A negative sign of the rating indicates that the result is smaller than the mean If one of the paired test results from a laboratory is rejected as an outlier based on the analysis, the other test result is still subjected to the analysis and is only rejected if it also fails to meet the outlier criteria.

3.3 OUTPUT OPTIONS

3.3.1 Reports

Various options are allowed in the program to generate and print information that is required for checking the input test results, review of the statistical data from the analyses, and printing of individual laboratory results for preliminary and final reports. The report options are accessible from the main screen through Report/Option in question (e.g. Report/Specified Lab). The options available are as follows:

Lab Mailing: This option basically generates the data sheet for reporting test results by the participants, and is accessible through Report/Lab Mailing. The data sheet generated includes the contact name, address, and phone number of the participating laboratory. We amend or modify the data sheet and the instructions for performing the tests each year, depending on the correlation samples distributed. In addition, the results reporting sheet with the participant's address may complicate the sample delivery. For these reasons, this option may not be useful.

Large Mail out: This option can be accessed through Report/ Large Mail out. This also generates the datasheet for reporting test results similar to that of "Lab Mailing", but the datasheet will not include contact name, address or any other information pertaining to the individual laboratories.

Preliminary: This option is accessible through Report/Preliminary and allows generating a report for each participant in the program. When the option is selected, the program will prompt with a statement (**This should take: 10 – 15 seconds. Do you wish to continue**), and "**Yes**" and "**No**" keys. If you want to generate the report, click "Yes" and use File/Print for sending the report to the printer. The report generated by this option will contain only the test results reported by the participating laboratory, and the report may be used to check any translation errors.

Lab Results: The access to this option is also similar to others under Report, and when selected, the program will prompt with the same statement as the one under Preliminary. This option allows generating and printing of results of the individual labs. Only the mean, standardised deviate and ratings for the tests in which a lab has participated will be provided in the Lab Results report. The participants use the preliminary or final report as a proof of participation in the correlation program. The preliminary or final report that we mail to participants consists of four pages, and will indicate all the tests for which the participant has submitted results as well as the tests that are available to participate. For these reasons, the report generated by this option may not be useful for mailing purposes.

Table of Test Results: This option allows generating and printing results of all the tests at once, and the report will include all the statistical data and ratings for the labs that have participated in each test. The operation of this option is similar to that of "Preliminary" and

"Lab Results" under Report.

Participants List: When this option is selected, the program will display the list of participants with a check mark in the boxes for those tests they have submitted results. The screen will also display a key "**Export to Excel**", and by clicking this key, the list can be exported and saved as Microsoft Excel spreadsheet.

Laboratory List: The information such as name of the contact person, address, phone and fax numbers of all the laboratories in the database can be viewed on the screen and printed by the use of this option. The lab number cannot be accessed through this option.

Test List: This option will provide all the title of the tests in the database, and category and coding numbers assigned to each of the test method. The category numbers are assigned by the HBSASB and they should be contacted for addition of new tests.

Specified Lab: This option is accessible through Report/Specified Lab and allows generating preliminary or final report for each participant in the program. When the option is selected, the program will prompt with a statement (**Please Select the Laboratory Number**), and "**OK**" and "**Cancel**" keys. To select the laboratory number, click on the arrow and scroll down the cursor to locate the laboratory by its number or name. If you want to generate the report, click "OK" and use File/Print for sending the report to the printer. The report generated by this option will be four pages long, and show the complete list of tests and the results submitted by the laboratory. The mean, standardised deviate and the rating obtained by the lab in each test are also included in this report.

Specified Test: The operation of this option is similar to the "Specified Lab", but the computer prompt will ask for the test number, instead of the lab number. The report will contain the lab number, test results of samples 1 and 2, standardised deviate, ratings, outlier labs, and the statistical data.

3.3.2 Scatter Plot

The program automatically generates a scatter plot at the end of the analysis and it can be viewed on the screen by clicking the icon, "**View Graph**", located within the outlier control. The scatter plot displayed will have the following information:

- Name of the test and the corresponding test number;
- Statistical data from the analysis employed; and
- Lab number of the laboratories that are identified as outliers.

In addition to the information listed above, the plot will display an **"MSWord"** button at the bottom of the plot area. By the click of this button, the graph can be converted to a Microsoft Word document and saved in a file or printed by the use of File/Print command. An example of the graph created in Microsoft Word is shown on Figure 4.

Sometimes, the outlier results may include values that vary from the mean by a wide margin, and it may cause difficulty to select the scale of the axes automatically. For these reasons, it is not practical to develop a computer program to accommodate all the outlier results within the plot. Therefore, the test results from the outlier laboratories will not be included in the plot generated at the end of the analysis. However, provisions are provided in the program to create a scatter plot as shown on Figure 5. Scatter plots identifying the outlier labs can be generated for each test as follows:

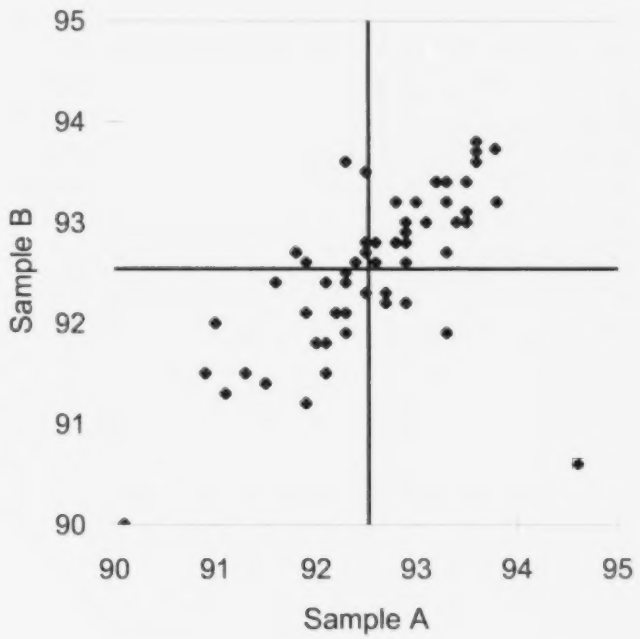
- At the end of the analysis, export the results using the icon, "**Export to Excel**", within the outlier control and save as a Microsoft Excel Spreadsheet;
- The exported Excel spreadsheet will contain the data as shown in Table 1;
- Call an old scatter plot file generated using "**Grapher**" plot program;
- Call the data file exported at the end of the analysis;
- Adjust the axis scales to fit the plot as shown on Figure 5;
- The test results of outlier labs may have to be adjusted to fit them within the plot area. To accommodate them within the plot area, call the data file and alter the values in Sample1and/or Sample 2 columns for the outlier labs in question. Make sure that the altered values fall within the axis scale selected.

Table 1 Data for Test No. 42 – Percent Passing the 75 µm Sieve

Set No.	Lab No.	Test No.	Sample 1	Sample 2	Stadized Dev1	Stadized Dev2	Rating 1	Rating 2	Outlier
2003	12	42	93.300	93.200	0.863	0.841	5	5	
2003	13	42	91.800	92.700	-0.960	0.208	-5	5	
2003	15	42	93.500	93.000	1.107	0.588	4	5	
2003	17	42	92.300	92.500	-0.352	-0.046	-5	-5	
2003	18	42	93.300	93.200	0.863	0.841	5	5	
2003	19	42	92.500	92.800	-0.109	0.334	-5	5	
2003	20	42	92.500	92.700	-0.109	0.208	-5	5	
2003	22	42	91.500	91.400	-1.325	-1.439	-4	-4	
2003	23	42	92.600	92.600	0.013	0.081	5	5	
2003	28	42	92.300	92.100	-0.352	-0.552	-5	-5	
2003	29	42	93.300	92.700	0.863	0.208	5	5	
2003	30	42	92.600	92.800	0.013	0.334	5	5	
2003	31	42	91.900	92.100	-0.838	-0.552	-5	-5	
2003	32	42	93.000	93.200	0.499	0.841	5	5	
2003	34	42	91.000	92.000	-1.932	-0.679	-3	-5	
2003	35	42	92.300	93.600	-0.352	1.348	-5	4	
2003	37	42	93.790	93.730	1.459	1.512	4	3	
2003	38	42	91.000	83.000	-1.932	-12.080	-3	0	38
2003	44	42	87.300	87.800	-6.430	-5.999	0	0	44
2003	46	42	92.700	92.200	0.134	-0.426	5	-5	
2003	47	42	93.600	93.800	1.228	1.601	4	3	

Set No.	Lab No.	Test No.	Sample 1	Sample 2	Stadized Dev1	Stadized Dev2	Rating 1	Rating 2	Outlier
2003	54	42	90.900	91.500	-2.054	-1.312	-2	-4	
2003	56	42	91.100	91.300	-1.811	-1.566	-3	-3	
2003	58	42	92.900	92.600	0.377	0.081	5	5	
2003	59	42	92.300	92.400	-0.352	-0.172	-5	-5	
2003	64	42	93.800	93.200	1.471	0.841	4	5	
2003	66	42	90.400	88.400	-2.662	-5.239	-1	0	66
2003	68	42	92.800	93.200	0.256	0.841	5	5	
2003	69	42	92.300	91.900	-0.352	-0.806	-5	-5	
2003	71	42	92.000	91.800	-0.717	-0.932	-5	-5	
2003	72	42	92.200	92.100	-0.474	-0.552	-5	-5	
2003	79	42	93.500	93.400	1.107	1.094	4	4	
2003	80	42	92.100	92.400	-0.595	-0.172	-5	-5	
2003	83	42	92.500	92.300	-0.109	-0.299	-5	-5	
2003	86	42	93.300	91.900	0.863	-0.806	5	-5	
2003	94	42	92.800	92.800	0.256	0.334	5	5	
2003	101	42	93.600	93.600	1.228	1.348	4	4	
2003	102	42	92.400	92.600	-0.231	0.081	-5	5	
2003	105	42	91.900	92.600	-0.838	0.081	-5	5	
2003	108	42	88.500	80.000	-4.971	-15.880	0	0	108
2003	112	42	92.500	93.500	-0.109	1.221	-5	4	
2003	119	42	92.400	92.600	-0.231	0.081	-5	5	
2003	120	42	92.100	91.800	-0.595	-0.932	-5	-5	
2003	138	42	92.900	92.200	0.377	-0.426	5	-5	
2003	139	42	92.900	92.900	0.377	0.461	5	5	
2003	146	42	92.900	92.800	0.377	0.334	5	5	
2003	148	42	94.600	90.600	2.444	-2.453	2	-2	
2003	172	42	92.700	92.300	0.134	-0.299	5	-5	
2003	175	42	91.600	92.400	-1.203	-0.172	-4	-5	
2003	176	42	84.600	76.600	-9.712	-20.187	0	0	176
2003	188	42	92.400	92.600	-0.231	0.081	-5	5	
2003	208	42	93.300	93.400	0.863	1.094	5	4	
2003	210	42	93.600	93.700	1.228	1.474	4	4	
2003	216	42	92.000	91.800	-0.717	-0.932	-5	-5	
2003	253	42	93.400	93.000	0.985	0.588	5	5	
2003	260	42	90.100	90.000	-3.026	-3.213	0	0	
2003	276	42	93.200	93.400	0.742	1.094	5	4	
2003	284	42	93.500	93.100	1.107	0.714	4	5	
2003	287	42	92.100	91.500	-0.595	-1.312	-5	-4	
2003	292	42	91.900	91.200	-0.838	-1.692	-5	-3	
2003	295	42	92.900	93.000	0.377	0.588	5	5	
2003	299	42	91.300	91.500	-1.568	-1.312	-3	-4	
2003	300	42	93.200	93.400	0.742	1.094	5	4	
2003	301	42	88.500	86.700	-4.971	-7.393	0	0	301
2003	306	42	93.100	93.000	0.620	0.588	5	5	

2003 MTO AGGREGATE AND SOIL PROFICIENCY SAMPLE TESTING PROGRAM



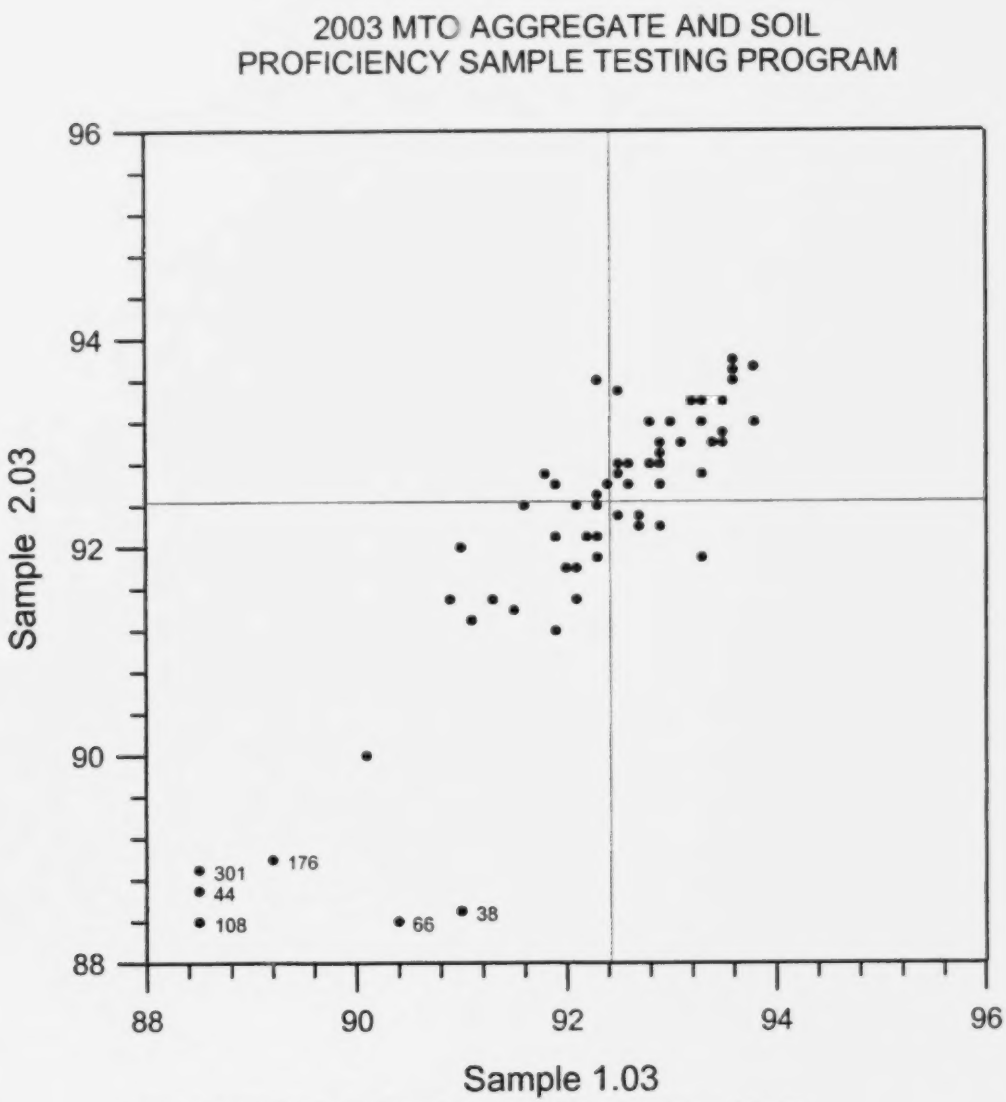
Test 42: Percent Passing the 75 um Sieve (Soil)

	Mat 1	Mat 2
Mean	92.528	92.536
Median	92.350	91.900
Std Dev	0.8788	0.7894

n = 59

Labs Eliminated: 38; 44; 66; 108; 176; 301;

Figure 4 Scatter Plot for Test No. 42



Test 42: Percent Passing the 75 μm Sieve (Soil)

	Mat 1	Mat 2
Mean	92.528	92.536
Median	92.550	88.700
Std Dev	0.879	0.789
n = 59		

Labs eliminated: 38, 44, 66, 108, 176, 301

Figure 5 Scatter Plot for Test No. 42

4.0 Acknowledgments

The authors wish to acknowledge the work of the Highways and Business Service Application Solutions Branch (HBSASB) of the Ministry of Transportation for developing the SASTP program. The assistance provided by Mr. Victor Mata of Bituminous Section in assisting in preparation of this manual is also gratefully acknowledged.

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Appendix A: Computation of Critical Values

The table below provides the calculation procedures for critical values at 95% confidence level, i.e., significance of 5%. Two hundred and fifteen laboratories participate in our Proficiency Sample Testing Program on an annual basis, even though 317 laboratories are listed in our database. The critical values calculations up to 300 observations will be adequate for all practical purposes, but provision is made in the SASTP program to handle up to 500 hundred observations.

Number of Observations = [N]
Significance level, α = 5%; $\Delta\alpha = [0.05/(2N)]$
Degrees of Freedom = [N – 2]
Probabilities for t Distribution from Microsoft Excel = t

$$\text{Critical Value } G_N = \frac{(N-1)}{\sqrt{N}} \sqrt{\frac{t^2_{(\alpha/2N),N-2)}}{N-2+t^2_{(\alpha/(2N),N-2)}}}$$

Critical values G_N for one-sided test, when the standard deviation is calculated from the same sample population.

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
6	0.004167	4	2.041	5.885	6.216	1.933
7	0.003571	5	2.268	5.164	5.628	2.081
8	0.003125	6	2.475	4.761	5.354	2.201
9	0.002778	7	2.667	4.506	5.225	2.300
10	0.002500	8	2.846	4.334	5.175	2.383
11	0.002273	9	3.015	4.210	5.170	2.455
12	0.002083	10	3.175	4.118	5.192	2.519
13	0.001923	11	3.328	4.048	5.233	2.574
14	0.001786	12	3.474	3.992	5.286	2.624
15	0.001667	13	3.615	3.948	5.347	2.669
16	0.001563	14	3.750	3.913	5.414	2.710
17	0.001471	15	3.881	3.883	5.485	2.748
18	0.001389	16	4.007	3.859	5.558	2.782
19	0.001316	17	4.129	3.839	5.633	2.814
20	0.001250	18	4.249	3.821	5.710	2.843
21	0.001190	19	4.364	3.807	5.787	2.871
22	0.001136	20	4.477	3.795	5.865	2.897
23	0.001087	21	4.587	3.784	5.943	2.921
24	0.001042	22	4.695	3.775	6.021	2.944
25	0.001000	23	4.800	3.768	6.099	2.965
26	0.000962	24	4.903	3.761	6.176	2.986

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
27	0.000926	25	5.004	3.755	6.253	3.005
28	0.000893	26	5.103	3.751	6.330	3.023
29	0.000862	27	5.199	3.747	6.406	3.041
30	0.000833	28	5.295	3.743	6.482	3.058
31	0.000806	29	5.388	3.740	6.557	3.074
32	0.000781	30	5.480	3.738	6.631	3.089
33	0.000758	31	5.570	3.735	6.705	3.104
34	0.000735	32	5.659	3.734	6.778	3.118
35	0.000714	33	5.747	3.732	6.851	3.131
36	0.000694	34	5.833	3.731	6.923	3.144
37	0.000676	35	5.918	3.731	6.994	3.157
38	0.000658	36	6.002	3.730	7.065	3.169
39	0.000641	37	6.085	3.729	7.135	3.180
40	0.000625	38	6.166	3.729	7.205	3.192
41	0.000610	39	6.247	3.729	7.274	3.203
42	0.000595	40	6.326	3.729	7.342	3.213
43	0.000581	41	6.405	3.729	7.410	3.224
44	0.000568	42	6.482	3.730	7.477	3.234
45	0.000556	43	6.559	3.730	7.544	3.243
46	0.000543	44	6.635	3.731	7.610	3.253
47	0.000532	45	6.710	3.731	7.676	3.262
48	0.000521	46	6.784	3.732	7.741	3.271
49	0.000510	47	6.857	3.733	7.806	3.279
50	0.000500	48	6.930	3.734	7.870	3.288
51	0.000490	49	7.001	3.735	7.934	3.296
52	0.000481	50	7.072	3.736	7.997	3.304
53	0.000472	51	7.143	3.737	8.060	3.312
54	0.000463	52	7.212	3.738	8.122	3.319
55	0.000455	53	7.281	3.739	8.184	3.327
56	0.000446	54	7.350	3.740	8.246	3.334
57	0.000439	55	7.417	3.742	8.307	3.341
58	0.000431	56	7.484	3.743	8.367	3.348
59	0.000424	57	7.551	3.744	8.427	3.355
60	0.000417	58	7.617	3.745	8.487	3.361
61	0.000410	59	7.682	3.746	8.546	3.368
62	0.000403	60	7.747	3.748	8.605	3.374
63	0.000397	61	7.811	3.749	8.663	3.380
64	0.000391	62	7.875	3.750	8.722	3.386
65	0.000385	63	7.938	3.752	8.779	3.392
66	0.000379	64	8.001	3.753	8.837	3.398
67	0.000373	65	8.063	3.754	8.894	3.404
68	0.000368	66	8.125	3.756	8.950	3.410
69	0.000362	67	8.186	3.757	9.007	3.415
70	0.000357	68	8.247	3.759	9.062	3.421
71	0.000352	69	8.307	3.760	9.118	3.426
72	0.000347	70	8.367	3.761	9.173	3.431
73	0.000342	71	8.427	3.763	9.228	3.436
74	0.000338	72	8.486	3.764	9.283	3.441

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
75	0.000333	73	8.545	3.766	9.337	3.446
76	0.000329	74	8.603	3.767	9.391	3.451
77	0.000325	75	8.661	3.769	9.445	3.456
78	0.000321	76	8.719	3.770	9.498	3.461
79	0.000316	77	8.776	3.772	9.551	3.466
80	0.000313	78	8.832	3.773	9.604	3.470
81	0.000309	79	8.889	3.775	9.657	3.475
82	0.000305	80	8.945	3.776	9.709	3.479
83	0.000301	81	9.001	3.778	9.761	3.484
84	0.000298	82	9.056	3.779	9.812	3.488
85	0.000294	83	9.111	3.781	9.864	3.492
86	0.000291	84	9.166	3.782	9.915	3.496
87	0.000287	85	9.220	3.783	9.965	3.500
88	0.000284	86	9.274	3.785	10.016	3.504
89	0.000281	87	9.328	3.786	10.066	3.508
90	0.000278	88	9.381	3.788	10.117	3.512
91	0.000275	89	9.435	3.789	10.166	3.516
92	0.000272	90	9.487	3.790	10.216	3.520
93	0.000269	91	9.540	3.792	10.265	3.524
94	0.000266	92	9.592	3.793	10.314	3.527
95	0.000263	93	9.644	3.795	10.363	3.531
96	0.000260	94	9.696	3.796	10.412	3.535
97	0.000258	95	9.747	3.797	10.460	3.539
98	0.000255	96	9.798	3.799	10.509	3.542
99	0.000253	97	9.849	3.800	10.556	3.545
100	0.000250	98	9.900	3.802	10.604	3.549
101	0.000248	99	9.950	3.803	10.652	3.552
102	0.000245	100	10.000	3.804	10.699	3.556
103	0.000243	101	10.050	3.806	10.746	3.559
104	0.000240	102	10.100	3.807	10.793	3.562
105	0.000238	103	10.149	3.808	10.840	3.565
106	0.000236	104	10.199	3.810	10.886	3.569
107	0.000234	105	10.247	3.811	10.933	3.572
108	0.000231	106	10.296	3.813	10.979	3.575
109	0.000229	107	10.345	3.814	11.025	3.578
110	0.000227	108	10.393	3.815	11.070	3.581
111	0.000225	109	10.441	3.816	11.116	3.584
112	0.000223	110	10.489	3.817	11.161	3.587
113	0.000221	111	10.536	3.819	11.206	3.591
114	0.000219	112	10.583	3.820	11.251	3.593
115	0.000217	113	10.631	3.821	11.296	3.596
116	0.000216	114	10.677	3.823	11.341	3.599
117	0.000214	115	10.724	3.824	11.385	3.602
118	0.000212	116	10.771	3.825	11.430	3.605
119	0.000210	117	10.817	3.827	11.474	3.608
120	0.000208	118	10.863	3.828	11.517	3.610
121	0.000207	119	10.909	3.829	11.561	3.613
122	0.000205	120	10.955	3.830	11.605	3.616

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
123	0.000203	121	11.000	3.832	11.648	3.619
124	0.000202	122	11.046	3.833	11.692	3.621
125	0.000200	123	11.091	3.834	11.735	3.624
126	0.000198	124	11.136	3.835	11.778	3.626
127	0.000197	125	11.181	3.837	11.820	3.629
128	0.000195	126	11.225	3.838	11.863	3.632
129	0.000194	127	11.270	3.839	11.905	3.634
130	0.000192	128	11.314	3.841	11.948	3.637
131	0.000191	129	11.358	3.842	11.990	3.639
132	0.000189	130	11.402	3.843	12.032	3.642
133	0.000188	131	11.446	3.844	12.074	3.644
134	0.000187	132	11.489	3.845	12.116	3.647
135	0.000185	133	11.533	3.846	12.157	3.649
136	0.000184	134	11.576	3.848	12.199	3.651
137	0.000182	135	11.619	3.849	12.240	3.654
138	0.000181	136	11.662	3.850	12.281	3.656
139	0.000180	137	11.705	3.851	12.322	3.658
140	0.000179	138	11.748	3.852	12.363	3.660
141	0.000177	139	11.790	3.853	12.404	3.663
142	0.000176	140	11.832	3.855	12.444	3.665
143	0.000175	141	11.875	3.856	12.485	3.667
144	0.000174	142	11.917	3.857	12.525	3.670
145	0.000172	143	11.959	3.858	12.565	3.672
146	0.000171	144	12.000	3.859	12.605	3.674
147	0.000170	145	12.042	3.860	12.645	3.676
148	0.000169	146	12.083	3.861	12.685	3.678
149	0.000168	147	12.125	3.863	12.725	3.680
150	0.000167	148	12.166	3.864	12.764	3.683
151	0.000166	149	12.207	3.865	12.804	3.685
152	0.000164	150	12.248	3.866	12.843	3.687
153	0.000163	151	12.288	3.867	12.882	3.689
154	0.000162	152	12.329	3.868	12.921	3.691
155	0.000161	153	12.370	3.869	12.960	3.693
156	0.000160	154	12.410	3.870	12.999	3.694
157	0.000159	155	12.450	3.871	13.038	3.696
158	0.000158	156	12.490	3.872	13.076	3.698
159	0.000157	157	12.530	3.873	13.115	3.700
160	0.000156	158	12.570	3.874	13.153	3.702
161	0.000155	159	12.610	3.875	13.192	3.705
162	0.000154	160	12.649	3.877	13.230	3.707
163	0.000153	161	12.689	3.878	13.268	3.709
164	0.000152	162	12.728	3.879	13.306	3.711
165	0.000152	163	12.767	3.880	13.344	3.713
166	0.000151	164	12.806	3.881	13.381	3.714
167	0.000150	165	12.845	3.881	13.419	3.715
168	0.000149	166	12.884	3.882	13.456	3.717
169	0.000148	167	12.923	3.884	13.494	3.719
170	0.000147	168	12.962	3.885	13.531	3.721

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
171	0.000146	169	13.000	3.886	13.568	3.723
172	0.000145	170	13.039	3.887	13.605	3.725
173	0.000145	171	13.077	3.888	13.643	3.727
174	0.000144	172	13.115	3.889	13.679	3.728
175	0.000143	173	13.153	3.889	13.716	3.730
176	0.000142	174	13.191	3.891	13.753	3.732
177	0.000141	175	13.229	3.892	13.789	3.734
178	0.000140	176	13.267	3.893	13.826	3.735
179	0.000140	177	13.304	3.894	13.862	3.737
180	0.000139	178	13.342	3.895	13.899	3.739
181	0.000138	179	13.379	3.895	13.935	3.740
182	0.000137	180	13.417	3.896	13.971	3.742
183	0.000137	181	13.454	3.898	14.007	3.744
184	0.000136	182	13.491	3.899	14.043	3.746
185	0.000135	183	13.528	3.900	14.079	3.747
186	0.000134	184	13.565	3.901	14.114	3.749
187	0.000134	185	13.602	3.901	14.150	3.750
188	0.000133	186	13.638	3.902	14.185	3.752
189	0.000132	187	13.675	3.903	14.221	3.754
190	0.000132	188	13.712	3.905	14.256	3.755
191	0.000131	189	13.748	3.906	14.292	3.757
192	0.000130	190	13.784	3.906	14.327	3.758
193	0.000130	191	13.820	3.907	14.362	3.760
194	0.000129	192	13.857	3.908	14.397	3.761
195	0.000128	193	13.893	3.909	14.432	3.763
196	0.000128	194	13.929	3.910	14.467	3.765
197	0.000127	195	13.964	3.910	14.501	3.766
198	0.000126	196	14.000	3.912	14.536	3.767
199	0.000126	197	14.036	3.913	14.571	3.769
200	0.000125	198	14.071	3.914	14.605	3.771
201	0.000124	199	14.107	3.914	14.640	3.771
202	0.000124	200	14.142	3.915	14.674	3.773
203	0.000123	201	14.178	3.916	14.708	3.775
204	0.000123	202	14.213	3.917	14.743	3.777
205	0.000122	203	14.248	3.919	14.777	3.778
206	0.000121	204	14.283	3.919	14.811	3.779
207	0.000121	205	14.318	3.920	14.845	3.781
208	0.000120	206	14.353	3.921	14.879	3.782
209	0.000120	207	14.388	3.922	14.912	3.784
210	0.000119	208	14.422	3.922	14.946	3.785
211	0.000118	209	14.457	3.923	14.980	3.786
212	0.000118	210	14.492	3.924	15.013	3.788
213	0.000117	211	14.526	3.926	15.047	3.790
214	0.000117	212	14.560	3.926	15.080	3.790
215	0.000116	213	14.595	3.927	15.114	3.792
216	0.000116	214	14.629	3.928	15.147	3.794
217	0.000115	215	14.663	3.928	15.180	3.794

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
218	0.000115	216	14.697	3.929	15.213	3.796
219	0.000114	217	14.731	3.930	15.246	3.797
220	0.000114	218	14.765	3.931	15.279	3.799
221	0.000113	219	14.799	3.931	15.312	3.800
222	0.000113	220	14.833	3.933	15.345	3.801
223	0.000112	221	14.866	3.934	15.378	3.803
224	0.000112	222	14.900	3.935	15.410	3.804
225	0.000111	223	14.933	3.935	15.443	3.805
226	0.000111	224	14.967	3.936	15.476	3.807
227	0.000110	225	15.000	3.937	15.508	3.808
228	0.000110	226	15.033	3.937	15.540	3.809
229	0.000109	227	15.067	3.938	15.573	3.810
230	0.000109	228	15.100	3.939	15.605	3.812
231	0.000108	229	15.133	3.939	15.637	3.812
232	0.000108	230	15.166	3.941	15.669	3.814
233	0.000107	231	15.199	3.942	15.702	3.816
234	0.000107	232	15.232	3.942	15.733	3.816
235	0.000106	233	15.264	3.943	15.765	3.818
236	0.000106	234	15.297	3.944	15.797	3.819
237	0.000105	235	15.330	3.944	15.829	3.820
238	0.000105	236	15.362	3.945	15.861	3.821
239	0.000105	237	15.395	3.946	15.893	3.823
240	0.000104	238	15.427	3.946	15.924	3.823
241	0.000104	239	15.460	3.948	15.956	3.825
242	0.000103	240	15.492	3.949	15.987	3.826
243	0.000103	241	15.524	3.949	16.019	3.827
244	0.000102	242	15.556	3.950	16.050	3.829
245	0.000102	243	15.589	3.951	16.081	3.830
246	0.000102	244	15.621	3.951	16.112	3.831
247	0.000101	245	15.653	3.952	16.144	3.832
248	0.000101	246	15.685	3.953	16.175	3.834
249	0.000100	247	15.716	3.953	16.206	3.834
250	0.000100	248	15.748	3.955	16.237	3.836
251	0.000100	249	15.780	3.956	16.268	3.837
252	0.000099	250	15.812	3.956	16.299	3.838
253	0.000099	251	15.843	3.957	16.330	3.839
254	0.000098	252	15.875	3.958	16.361	3.841
255	0.000098	253	15.906	3.958	16.391	3.841
256	0.000098	254	15.938	3.959	16.422	3.843
257	0.000097	255	15.969	3.960	16.453	3.844
258	0.000097	256	16.000	3.960	16.483	3.844
259	0.000097	257	16.031	3.962	16.513	3.846
260	0.000096	258	16.062	3.963	16.544	3.847
261	0.000096	259	16.094	3.963	16.574	3.848
262	0.000095	260	16.125	3.964	16.605	3.849
263	0.000095	261	16.156	3.965	16.635	3.851
264	0.000095	262	16.187	3.965	16.665	3.851
265	0.000094	263	16.217	3.965	16.695	3.852

[N]	[0.05/(2N)]	[N-2]	(N-1)/sqrt(N)	t	sqrt(N - 2 + t^2)	G_N
266	*0.000094*	*264*	*16.248*	*3.966*	*16.725*	**3.853**
267	*0.000094*	*265*	*16.279*	*3.967*	*16.755*	**3.855**
268	*0.000093*	*266*	*16.310*	*3.967*	*16.785*	**3.855**
269	*0.000093*	*267*	*16.340*	*3.969*	*16.815*	**3.857**
270	*0.000093*	*268*	*16.371*	*3.970*	*16.845*	**3.858**
271	*0.000092*	*269*	*16.401*	*3.970*	*16.875*	**3.858**
272	*0.000092*	*270*	*16.432*	*3.971*	*16.905*	**3.860**
273	*0.000092*	*271*	*16.462*	*3.972*	*16.935*	**3.861**
274	*0.000091*	*272*	*16.493*	*3.972*	*16.964*	**3.862**
275	*0.000091*	*273*	*16.523*	*3.972*	*16.993*	**3.862**
276	*0.000091*	*274*	*16.553*	*3.974*	*17.023*	**3.865**
277	*0.000090*	*275*	*16.583*	*3.974*	*17.053*	**3.865**
278	*0.000090*	*276*	*16.613*	*3.974*	*17.082*	**3.865**
279	*0.000090*	*277*	*16.643*	*3.976*	*17.112*	**3.867**
280	*0.000089*	*278*	*16.673*	*3.977*	*17.141*	**3.868**
281	*0.000089*	*279*	*16.703*	*3.977*	*17.170*	**3.869**
282	*0.000089*	*280*	*16.733*	*3.978*	*17.200*	**3.870**
283	*0.000088*	*281*	*16.763*	*3.979*	*17.229*	**3.872**
284	*0.000088*	*282*	*16.793*	*3.979*	*17.258*	**3.872**
285	*0.000088*	*283*	*16.823*	*3.979*	*17.287*	**3.872**
286	*0.000087*	*284*	*16.852*	*3.980*	*17.316*	**3.874**
287	*0.000087*	*285*	*16.882*	*3.981*	*17.345*	**3.875**
288	*0.000087*	*286*	*16.912*	*3.981*	*17.374*	**3.875**
289	*0.000087*	*287*	*16.941*	*3.983*	*17.403*	**3.877**
290	*0.000086*	*288*	*16.971*	*3.984*	*17.432*	**3.878**
291	*0.000086*	*289*	*17.000*	*3.984*	*17.461*	**3.879**
292	*0.000086*	*290*	*17.029*	*3.984*	*17.489*	**3.879**
293	*0.000085*	*291*	*17.059*	*3.985*	*17.518*	**3.880**
294	*0.000085*	*292*	*17.088*	*3.986*	*17.547*	**3.882**
295	*0.000085*	*293*	*17.117*	*3.986*	*17.575*	**3.882**
296	*0.000084*	*294*	*17.147*	*3.987*	*17.604*	**3.884**
297	*0.000084*	*295*	*17.176*	*3.988*	*17.633*	**3.885**
298	*0.000084*	*296*	*17.205*	*3.988*	*17.661*	**3.885**
299	*0.000084*	*297*	*17.234*	*3.988*	*17.689*	**3.886**
300	*0.000083*	*298*	*17.263*	*3.990*	*17.718*	**3.887**

Appendix B: Probabilities for the Student 't' Distribution

Each table entry is the value (Y) of the random variable $Y \sim t_v$ such that $P_r(Y \leq y)$ equal to the probability given in the column heading.
D_f – Degrees of Freedom

Df (v)	0.80	0.85	0.90	0.95	0.975	0.99	0.995	0.999	0.999 5	0.999 9	0.999 95	0.999 99	0.999 995	0.999 999
1	1.37 638	1.96 261	3.07 768	6.31 375	12.7 062	31.8 205	63.6 567	318. 309	636. 619	3183. 10	6366. 20	31 831.0	63 662.0	318 309.
2	1.06 066	1.38 621	1.88 562	2.91 999	4.30 265	6.96 456	9.92 484	22.3 271	31.5 991	70.7 001	99.9 925	223. 603	316. 225	707. 106
3	.978 472	1.24 978	1.63 774	2.35 336	3.18 245	4.54 070	5.84 091	10.2 145	12.9 240	22.2 037	28.0 001	47.9 277	60.3 968	103. 299
4	.940 965	1.18 957	1.53 321	2.13 185	2.77 645	3.74 695	4.60 409	7.17 318	8.61 030	13.0 337	15.5 441	23.3 322	27.7 716	41.5 779
5	.919 544	1.15 577	1.47 588	2.01 505	2.57 058	3.36 493	4.03 214	5.89 343	6.86 883	9.67 757	11.1 777	15.5 469	17.8 969	24.7 710
6	.905 703	1.13 416	1.43 976	1.94 318	2.44 691	3.14 267	3.70 743	5.20 763	5.95 882	8.02 479	9.08 235	12.0 317	13.5 553	17.8 303
7	.896 030	1.11 916	1.41 492	1.89 458	2.36 462	2.99 795	3.49 948	4.78 529	5.40 788	7.06 343	7.88 458	10.1 027	11.2 148	14.2 415
8	.888 890	1.10 815	1.39 682	1.85 955	2.30 600	2.89 646	3.35 539	4.50 079	5.04 131	6.44 220	7.12 000	8.90 703	9.78 253	12.1 098
9	.883 404	1.09 972	1.38 303	1.83 311	2.26 216	2.82 144	3.24 984	4.29 681	4.78 091	6.01 013	6.59 368	8.10 206	8.82 748	10.7 202
10	.879 058	1.09 306	1.37 218	1.81 246	2.22 814	2.76 377	3.16 927	4.14 370	4.58 689	5.69 382	6.21 105	7.52 695	8.15 029	9.75 200
11	.875 530	1.08 767	1.36 343	1.79 588	2.20 099	2.71 808	3.10 581	4.02 470	4.43 698	5.45 276	5.92 119	7.09 736	7.64 749	9.04 328
12	.872 609	1.08 321	1.35 622	1.78 229	2.17 881	2.68 100	3.05 454	3.92 963	4.31 779	5.26 327	5.69 447	6.76 517	7.26 063	8.50 430
13	.870 152	1.07 947	1.35 017	1.77 093	2.16 037	2.65 031	3.01 228	3.85 198	4.22 083	5.11 058	5.51 252	6.50 114	6.95 444	8.08 184
14	.868 055	1.07 628	1.34 503	1.76 131	2.14 479	2.62 449	2.97 684	3.78 739	4.14 045	4.98 501	5.36 341	6.28 655	6.70 645	7.74 251
15	.866 245	1.07 353	1.34 061	1.75 305	2.13 145	2.60 248	2.94 671	3.73 283	4.07 277	4.88 000	5 23 909	6.10 887	6.50 175	7.46 438
16	.864 667	1.07 114	1.33 676	1.74 588	2.11 991	2.58 349	2.92 078	3.68 615	4.01 500	4.79 091	5.13 389	5.95 944	6.33 005	7.23 254
17	.863 279	1.06 903	1.33 338	1.73 961	2.10 982	2.56 693	2.89 823	3.64 577	3.96 513	4.71 441	5.04 376	5.83 210	6.18 406	7.03 649
18	.862 049	1.06 717	1.33 039	1.73 406	2.10 092	2.55 238	2.87 844	3.61 048	3.92 165	4.64 801	4.96 571	5.72 233	6.05 848	6.86 864
19	.860 951	1.06 551	1.32 773	1.72 913	2.09 302	2.53 948	2.86 093	3.57 940	3.88 341	4.58 986	4.89 746	5.62 677	5.94 935	6.72 339
20	.859 964	1.06 402	1.32 534	1.72 472	2.08 596	2.52 798	2.84 534	3.55 181	3.84 952	4.53 852	4.83 730	5.54 284	5.85 367	6.59 653
21	.859 074	1.06 267	1.32 319	1.72 074	2.07 961	2.51 765	2.83 136	3.52 715	3.81 928	4.49 286	4.78 388	5.46 856	5.76 911	6.48 480
22	.858 266	1.06 145	1.32 124	1.71 714	2.07 387	2.50 832	2.81 876	3.50 499	3.79 213	4.45 199	4.73 612	5.40 237	5.69 386	6.38 568
23	.857 530	1.06 034	1.31 946	1.71 387	2.06 866	2.49 987	2.80 734	3.48 496	3.76 763	4.41 520	4.69 319	5.34 303	5.62 647	6.29 717
24	.856 855	1.05 932	1.31 784	1.71 088	2.06 390	2.49 216	2.79 694	3.46 678	3.74 540	4.38 192	4.65 438	5.28 952	5.56 578	6.21 766
25	.856 236	1.05 838	1.31 635	170 814	2.05 954	2.48 511	2.78 744	3.45 019	3.72 514	4.35 165	4.61 914	5.24 104	5.51 085	6.14 586

Df (v)	0.80	0.85	0.90	0.95	0.975	0.99	0.995	0.999	0.999 5	0.999 9	0.999 95	0.999 99	0.999 995	0.999 999
26	.855 665	1.05 752	1.31 497	1.70 562	2.05 553	2.47 863	2.77 871	3.43 500	3.70 661	4.32 402	4.58 698	5.19 692	5.46 089	6.08 072
27	.855 137	1.05 673	1.31 370	1.70 329	2.05 183	2.47 266	2.77 068	3.42 103	3.68 959	4.29 870	4.55 754	5.15 658	5.41 527	6.02 134
28	.854 647	1.05 599	1.31 253	1.70 113	2.04 841	2.46 714	2.76 326	3.40 816	3.67 391	4.27 540	4.53 047	5.11 958	5.37 345	5.96 701
29	.854 192	1.05 530	1.13 143	1.69 913	2.04 523	2.46 202	2.75 639	3.39 624	3.65 941	4.25 389	4.50 551	5.08 550	5.33 497	5.91 711
30	.853 767	1.05 466	1.31 042	1.69 726	2.04 227	2.45 726	2.75 000	3.38 518	3.64 596	4.23 399	4.48 242	5.05 403	5.29 945	5.87 112
31	.853 370	1.05 406	1.30 946	1.69 552	2.03 951	2.45 282	2.74 404	3.37 490	3.63 346	4.21 550	4.46 099	5.02 487	5.26 657	5.82 860
32	.852 998	1.05 350	1.30 857	1.69 389	2.03 693	2.44 868	2.73 848	3.36 531	3.62 180	4.19 830	4.44 105	4.99 779	5.23 603	5.78 918
33	.852 649	1.05 298	1.30 774	1.69 236	2.03 452	2.44 479	2.73 328	3.35 634	3.61 091	4.18 224	4.42 246	4.97 255	5.20 761	5.75 253
34	.852 321	1.05 248	1.30 695	1.69 092	2.03 224	2.44 115	2.72 839	3.34 793	3.60 072	4.16 722	4.40 508	4.94 899	5.18 108	5.71 838
35	.852 012	1.05 202	1.30 621	1.68 957	2.03 011	2.43 772	2.72 381	3.34 005	3.59 115	4.15 315	4.38 880	4.92 695	5.15 627	5.68 647
36	.851 720	1.05 158	1.30 551	1.68 830	2.02 809	2.43 449	2.71 948	3.33 262	3.58 215	4.13 993	4.37 351	4.90 627	5.13 302	5.65 659
37	.851 444	1.05 117	1.30 485	1.68 709	2.02 619	2.43 145	2.71 541	3.32 563	3.57 367	4.12 748	4.35 913	4.88 685	5.11 117	5.62 856
38	.851 183	1.05 077	1.30 423	1.68 595	2.02 439	2.42 857	2.71 156	3.31 903	3.56 568	4.11 576	4.34 558	4.86 856	5.09 062	5.60 220
39	.850 935	1.05 040	1.30 364	1.68 488	2.02 269	2.42 584	2.70 791	3.31 279	3.55 812	4.10 468	4.33 279	4.85 131	5.07 125	5.57 738
40	.850 700	1.05 005	1.30 308	1.68 385	2.02 108	2.42 326	2.70 446	3.30 688	3.55 097	4.09 421	4.32 070	4.83 502	5.05 295	5.55 397
41	.850 476	1.04 971	1.30 254	1.68 288	2.01 954	2.42 080	2.70 118	3.30 127	3.54 418	4.08 429	4.30 925	4.81 961	5.03 565	5.53 184
42	.850 263	1.04 939	1.30 204	1.68 195	2.01 808	2.41 847	2.69 807	3.29 595	3.53 775	4.07 488	4.29 839	4.80 501	5.01 926	5.51 090
43	.850 060	1.04 908	1.30 155	1.68 107	2.01 669	2.41 625	2.69 510	3.29 089	3.53 163	4.06 594	4.28 809	4.79 115	5.00 372	5.49 106
44	.849 867	1.04 879	1.30 109	1.68 023	2.01 537	2.41 413	2.69 228	3.28 607	3.52 580	4.05 744	4.27 829	4.77 799	4.98 896	5.47 222
45	.849 682	1.04 852	130 065	1.67 943	2.01 410	2.41 212	2.68 959	3.28 148	3.52 025	4.04 934	4.26 896	4.76 547	4.97 492	5.45 432
46	.849 505	1.04 825	1.30 023	1.67 866	2.01 290	2.41 019	2.68 701	3.27 710	3.51 496	4.04 162	4.26 006	4.75 354	4.96 155	5.43 728
47	.849 336	1.04 800	1.29 982	1.67 793	2.01 174	2.40 835	2.68 456	3.27 291	3.50 990	4.03 426	4.25 158	4.74 216	4.94 880	5.42 106
48	.849 174	1.04 775	1.29 944	1.67 722	2.01 063	2.40 658	2.68 220	3.26 891	3.50 507	4.02 722	4.24 347	4.73 131	4.93 664	5.40 558
49	.849 018	1.04 752	1.29 907	1.67 655	2.00 958	2.40 489	2.67 995	3.26 508	3.50 044	4.02 049	4.23 573	4.72 093	4.92 502	5.39 080
50	.848 869	1.04 729	1.29 871	1.67 591	2.00 856	2.40 327	2.67 779	3.26 141	3.49 601	4.01 405	4.22 831	4.71 101	4.91 391	5.37 667
52	.848 588	1.04 687	1.29 805	1.67 469	2.00 665	2.40 022	2.67 373	3.25 451	3.48 769	4.00 195	4.21 439	4.69 240	4.89 308	5.35 022
54	.848 328	1.04 648	1.29 743	1.67 356	2.00 488	2.39 741	2.66 998	3.24 815	3.48 002	3.99 081	4.20 158	4.67 528	4.87 392	5.32 591
56	.848 087	1.04 612	1.29 685	1.67 252	2.00 324	2.39 480	2.66 651	3.24 226	3.47 292	3.98 051	4.18 974	4.65 947	4.85 625	5.30 350
58	.847 862	1.04 578	1.29 632	1.67 155	2.00 172	2.39 238	2.66 329	3.23 680	3.46 633	3.97 096	4.17 876	4.64 484	4.83 988	5.28 278
60	.847 653	1.04 547	1.29 582	1.67 065	2.00 030	2.39 012	2.66 028	3.23 171	3.46 020	3.96 209	4.16 857	4.63 125	4.82 470	5.26 355
62	.847 457	1.04 518	1.29 536	1.66 980	1.99 897	2.38 801	2.65 748	3.22 696	3.45 448	3.95 382	4.15 906	4.61 860	4.81 056	5.24 568
64	.847 274	1.04 490	1.29 492	1.66 901	1.99 773	2.38 604	2.65 485	3.22 253	3.44 914	3.94 609	4.15 019	4.60 679	4.79 737	5.22 901

Df (v)	0.80	0.85	0.90	0.95	0.975	0.99	0.995	0.999	0.999 5	0.999 9	0.999 95	0.999 99	0.999 995	0.999 999
66	.847 101	1.04 464	1.29 451	1.66 827	1.99 656	2.38 419	2.65 239	3.21 837	3.44 414	3.93 885	4.14 188	4.59 575	4.78 504	5.21 343
68	.846 939	1.04 440	1.29 413	1.66 757	1.99 547	2.38 245	2.65 008	3.21 446	3.43 944	3.93 207	4.13 409	4.58 539	4.77 348	5.19 884
70	.846 786	1.04 417	1.29 376	1.66 691	1.99 444	2.38 081	2.64 790	3.21 079	3.43 501	3.92 568	4.12 677	4.57 567	4.76 262	5.18 514
72	.846 642	1.04 395	1.29 342	1.66 629	1.99 346	2.37 926	2.64 585	3.20 733	3.43 085	3.91 967	4.11 987	4.56 651	4.75 241	5.17 226
74	.846 506	1.04 375	1.29 310	1.66 571	1.99 254	2.37 780	2.64 391	3.20 406	3.42 692	3.91 400	4.11 337	4.55 788	4.74 278	5.16 013
76	.846 376	1.04 355	1.29 279	1.66 515	1.99 167	2.37 642	2.64 208	3.20 096	3.42 320	3.90 864	4.10 722	4.54 973	4.73 369	5.14 868
78	.846 254	1.04 337	1.29 250	1.66 462	1.99 085	2.37 511	2.64 034	3.19 804	3.41 968	3.90 357	4.10 141	4.54 202	4.72 509	5.13 785
80	.846 137	1.04 320	1.29 222	1.66 412	1.99 006	2.37 387	2.63 869	3.19 526	3.41 634	3.89 876	4.09 589	4.53 472	4.71 695	5.12 760
82	.846 027	1.04 303	1.29 196	1.66 365	1.98 932	2.37 269	2.63 712	3.19 262	3.41 317	3.89 419	4.09 066	4.52 779	4.70 922	5.11 789
84	.845 921	1.04 287	1.29 171	1.66 320	1.98 861	2.37 156	2.63 563	3.19 011	3.41 015	3.88 985	4.08 569	4.52 120	4.70 189	5.10 866
86	.845 821	1.04 272	1.29 147	1.66 277	1.98 793	2.37 049	2.63 421	3.18 772	3.40 728	3.88 573	4.08 096	4.51 494	4.69 491	5.09 989
88	.845 725	1.04 258	1.29 125	1.66 235	1.98 729	2.36 947	2.63 286	3.18 544	3.40 455	3.88 179	4.07 645	4.50 898	4.68 827	5.09 154
90	.845 633	1.04 244	1.29 103	1.66 196	1.98 667	2.36 850	2.63 157	3.18 327	3.40 194	3.87 804	4.07 216	4.50 330	4.68 193	5.08 359
92	.845 546	1.04 231	1.29 082	1.66 159	1.98 609	2.36 757	2.63 033	3.18 119	3.39 944	3.87 445	4.06 805	4.49 787	4.67 589	5.07 600
94	.845 462	1.04 218	1.29 062	1.66 123	1.98 552	2.36 667	2.62 915	3.17 921	3.39 706	3.87 103	4.06 413	4.49 268	4.67 012	5.06 875
96	.845 381	1.04 206	1.29 043	1.66 088	1.98 498	2.36 582	2.62 802	3.17 731	3.39 477	3.86 775	4.06 038	4.48 773	4.66 459	5.06 182
98	.845 304	1.04 195	1.29 025	1.66 055	1.98 447	2.36 500	2.62 693	3.17 549	3.39 259	3.86 461	4.05 678	4.48 298	4.65 931	5.05 518
99	.845 267	1.04 189	1.29 016	1.66 039	1.98 422	2.36 461	2.62 641	3.17 460	3.39 153	3.86 309	4.05 504	4.48 068	4.65 675	5.05 197
100	.845 230	1.04 184	1.29 007	1.66 023	1.98 397	2.36 422	2.62 589	3.17 374	3.39 049	3.86 160	4.05 334	4.47 843	4.65 424	5.04 883
105	.845 058	1.04 158	1.28 967	1.65 950	1.98 282	2.36 239	2.62 347	3.16 967	3.38 561	3.85 459	4.04 532	4.46 784	4.64 246	5.03 406
110	.844 901	1.04 134	1.28 930	1.65 882	1.98 177	2.36 073	2.62 126	3.16 598	3.38 118	3.84 824	4.03 805	4.45 825	4.63 179	5.02 069
115	.844 758	1.04 113	1.28 896	1.65 821	1.98 081	2.35 921	2.61 926	3.16 262	3.37 714	3.84 246	4.03 144	4.44 953	4.62 209	5.00 854
120	.844 627	1.04 093	1.28 865	1.65 765	1.97 993	2.35 782	2.61 742	3.15 954	3.37 345	3.83 717	4.02 539	4.44 156	4.61 322	4.99 744
125	.844 506	1.04 075	1.28 836	1.65 714	1.97 912	2.35 655	2.61 573	3.15 671	3.37 006	3.83 232	4.01 984	4.43 425	4.60 509	4.98 727
130	.844 395	1.04 058	1.28 810	1.65 666	1.97 838	2.35 537	2.61 418	3.15 411	3.36 694	3.82 785	4.01 473	4.42 752	4.59 761	4.97 792
135	.844 292	1.04 043	1.28 785	1.65 622	1.97 769	2.35 429	2.61 274	3.15 170	3.36 405	3.82 372	4.01 001	4.42 131	4.59 070	4.96 928
140	.844 196	1.04 029	1.28 763	1.65 581	1.97 705	2.35 328	2.61 140	3.14 947	3.36 138	3.81 989	4.00 563	4.41 555	4.58 430	4.96 128
145	.844 107	1.04 015	1.28 742	1.65 543	1.97 646	2.35 234	2.61 016	3.14 739	3.35 889	3.81 633	4.00 157	4.41 020	4.57 835	4.95 385
150	.844 024	1.04 003	1.28 722	1.65 508	1.97 591	2.35 146	2.60 900	3.14 545	3.35 657	3.81 301	3.99 778	4.40 522	4.57 282	4.94 694
160	.843 874	1.03 980	1.28 687	1.65 443	1.97 490	2.34 988	2.60 691	3.14 195	3.35 237	3.80 701	3.99 093	4.39 621	4.56 281	4.93 445
170	.843 741	1.03 960	1.28 655	1.65 387	1.97 402	2.34 848	2.60 506	3.13 886	3.34 868	3.80 174	3.98 490	4.38 830	4.55 402	4.92 347
180	.843 623	1.03 943	1.28 627	1.65 336	1.97 323	2.34 724	2.60 342	3.13 612	3.34 540	3.79 706	3.97 955	4.38 128	4.54 622	4.91 375

Df (v)	0.80	0.85	0.90	0.95	0.975	0.99	0.995	0.999	0.999 5	0.999 9	0.999 95	0.999 99	0.999 995	0.999 999
190	.843 517	1.03 927	1.28 602	1.65 291	1.97 253	2.34 613	2.60 195	3.13 368	3.34 247	3.79 288	3.97 478	4.37 501	4.53 927	4.90 508
200	.843 422	1.03 913	1.28 580	1.65 251	1.97 190	2.34 514	2.60 063	3.13 148	3.33 984	3.78 912	3.97 050	4.36 939	4.53 302	4.89 730
250	.843 061	1.03 859	1.28 495	1.65 097	1.96 950	2.34 136	2.59 564	3.12 315	3.32 987	3.77 491	3.95 428	4.34 813	4.50 943	4.86 793
300	.842 821	1.03 823	1.28 438	1.64 995	1.96 790	2.33 884	2.59 232	3.11 762	3.32 325	3.76 549	3.94 354	4.33 406	4.49 382	4.84 851
350	.842 649	1.03 797	1.28 398	1.64 922	1.96 677	2.33 705	2.58 995	3.11 368	3.31 854	3.75 879	3.93 590	4.32 406	4.48 273	4.83 473
400	.842 521	1.03 778	1.28 367	1.64 867	1.96 591	2.33 571	2.58 818	3.11 073	3.31 502	3.75 378	3.93 018	4.31 659	4.47 444	4.82 443
450	.842 421	1.03 763	1.28 344	1.64 825	1.96 525	2.33 466	2.58 680	3.10 844	3.31 228	3.74 989	3.92 575	4.31 079	4.46 802	4.81 645
500	.842 341	1.03 751	1.28 325	1.64 791	1.96 472	2.33 383	2.58 570	3.10 661	3.31 009	3.74 678	3.92 221	4.30 616	4.46 289	4.81 009
600	.842 221	1.03 733	1.28 296	1.64 740	1.96 393	2.33 258	2.58 405	3.10 387	3.30 682	3.74 213	3.91 691	4.29 923	4.45 521	4.80 056
700	.842 135	1.03 720	1.28 276	1.64 703	1.96 336	2.33 169	2.58 287	3.10 192	3.30 448	3.73 881	3.91 313	4.29 430	4.44 974	4.79 378
800	.842 071	1.03 711	1.28 261	1.64 676	1.96 293	2.33 102	2.58 199	3.10 045	3.30 273	3.73 632	3.91 030	4.29 060	4.44 565	4.78 870
900	.842 021	1.03 703	1.28 249	1.64 655	1.96 260	2.33 050	2.58 130	3.09 931	3.30 137	3.73 439	3.90 810	4.28 773	4.44 247	4.78 476
1000	.841 981	1.03 697	1.28 240	1.64 638	1.96 234	2.33 008	2.58 075	3.09 840	3.30 028	3.73 285	3.90 634	4.28 544	4.43 993	4.78 161
1500	.841 861	1.03 679	1.28 212	1.64 587	1.96 155	2.32 884	2.57 911	3.09 567	3.29 702	3.72 823	3.90 108	4.27 857	4.43 232	4.77 218
2000	.841 801	1.03 670	1.28 198	1.64 562	1.96 115	2.32 821	2.57 829	3.09 431	3.29 540	3.72 592	3.89 845	4.27 514	4.42 852	4.76 748
2500	.841 765	1.03 665	1.28 189	1.64 546	1.96 091	2.32 784	2.57 780	3.09 350	3.29 442	3.72 454	3.89 688	4.27 309	4.42 625	4.76 466
3000	.841 741	1.03 661	1.28 183	1.64 536	1.96 076	2.32 759	2.57 747	3.09 295	3.29 377	3.72 362	3.89 583	4.27 172	4.42 473	4.76 279
4000	.841 711	1.03 657	1.28 176	1.64 523	1.96 056	2.32 728	2.57 706	3.09 229	3.29 315	3.72 247	3.89 452	4.27 004	4.42 287	4.76 044
5000	.841 693	1.03 654	1.28 172	1.64 516	1.96 044	2.32 709	2.57 681	3.09 191	3.29 318	3.72 181	3.89 373	4.26 899	4.42 172	4.76 089
∞	.841 621	1.03 643	1.28 155	1.64 485	1.95 996	2.32 635	2.57 583	3.09 023	3.29 053	3.71 902	3.89 059	4.26 489	4.41 717	4.75 342

Appendix C: Fisher's F Distribution Values

The values Fd_{num}, d_{den}, p for probability $p = 0.95$ are provided in the table below. Where $Pr(F < Fd_{num}, d_{den}, p) = p$, and F has Fisher's F distribution with numerator degrees of freedom d_{num}, and denominator degrees of freedom d_{den}.

Probability p = 0.95

	d_{num}										
d_{den}	1	2	3	4	5	6	7	8	12	24	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	243.91	249.05	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.41	19.45	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	1.95	1.74	1.44
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.75	1.52	1.00